Euler’s Gamma Function:

Г: (0, ) (0, ), Г(a) =

* Г(1) = 1
* Г(a+1) = a Г(a)
* Г(n+1)= n!, n
* Г() =

Euler’s Beta Function:

: (0, )x(0, ) (0, ), (a, b) =

* (a, 1) = a
* (a, b) = (b, a)
* (a, b) = (a -1, b + 1)
* (a, b) = (a, b - 1) = (a – 1, b)
* (a, b) =

Arrangements:

Permutations:

Combinations:

De Morgan’s laws:

=

A : complementary event

A: event occurs if A or B or both occur

A: event occurs if A and B occur

A \ B = A : event occurs if A occurs and B does not

A = (A\B) (A\( A) :event occurs if A or B occur, but

not both

Classical Probability: P(A) =

Collectively Exhaustive: = S

Partition: = S

= , i, j∈ I, i ≠ j

Mutually Exclusive Events: P(A ) = 0

Rules of Probability: P() = 1- P(A)

P(A) = P(A) + P(B) - P(A)

P(A \ B) = P(A) - P(A)

Conditional Probability: P(A|B) =

Independent Events: P(A) = P(A)P(B)

P(A|B) = P(A)

Total Probability Rule: P(E) =

Multiplication Rule:

P() = P()P()P()…P()

Cumulative Distribution Function (Cdf): F(X) = P(X ) =

Joint Probability Distribution Functions

F(x, y) = P(X ) =

Marginal Densities

= P(X = ) =

= P(Y = ) =

X, Y

Independent events: = P(X = ) P(Y = )=

Scalar Multiplication: X

Sum: X+Y

Product:

Quotient: X/Y

X: continuous random variable; Pdf: f(x); Cdf: F(x)

* F(X) = P(X x) =
* = 1
* P(a < X < b) = P (a < X < b) =
* F(-) = 0, F() = 1

(X,Y):continuous random vector

Pdf: f(x, y),

Cdf: F(x, y)= P(X )=

* P() = F() - F() - F() +F()
* F() = 1, F(-) = F(x, -) = 0
* (x) = F(x, ), (x) = F(, y) – marginal cdf’s
* P((X, Y)) =
* (x) = ; (y) =
* X, Y- independent: (x, y) = (x) (y)

Function: Y = g(X): X, g: RR, g- strictly monotone, differentiable

g’(x) = 0

, y g(R)

Numerical Characteristics of Random Variables

Expectation: E(X) = for discrete variables

E(X) = for continuous variables

Variance: V(X) = E() = E() - =

Standard Deviation = Std(X) =

Moments:

* Moment of order k:
* Absolute Moment of order k:
* Central Moments of order k:

Covariance: cov(X, Y) = E((X-E(X))(Y – E(Y)))

Correlation Coefficient: (X, Y) =

Properties:

* E(aX+ b) = aE(X) + b; V(ax+b) = V(X)
* E(X+Y) = E(X) + E(Y)
* X,Y -independent: E(XY)=E(X)E(Y) and V(X+Y) =V(X)+V(Y)
* h:R R, h – measurable function

E(h(X)) = (X - discrete)

E(h(X)) = (X – continuous)

* cov(X, Y) = E(XY) -E(X)E(Y)
* X,Y -independent: cov(X, Y)=(X, Y) = 0
* V() = + 2
* -1≤(X, Y) ≤ 1; (X, Y) = ± 1 ⇔ ∃ a,b ∈R, a≠0 s.t. Y= aX+b

E(h(X,Y)) = , (X, Y)-cont. random vector